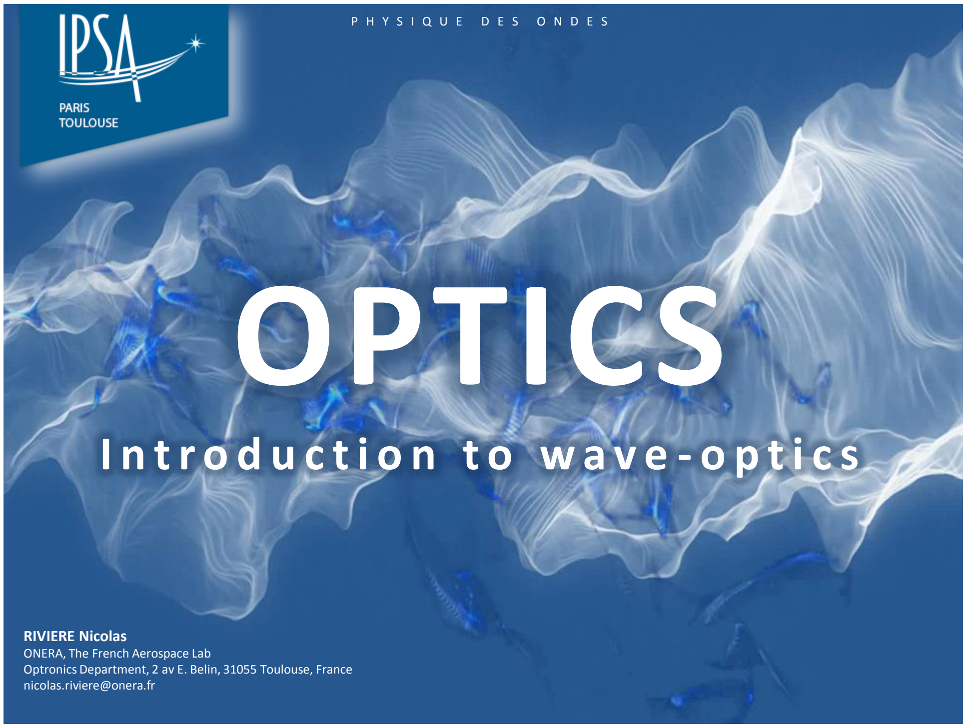
PARIS
TOULOUSE
PHYSIQUE DES ONDES
OPTICS
Introduction to wave-optics
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PARIS
TOULOUSE
Chapter
General context
In this chapter, we introduce the goal of this lecture and simple observations of light effects. The general context is focused on aerial and aerospace applications.

OPTICS FOR REMOTE SENSING

I. General context

OBJECTIVES

- Light has long captured the fascination of humankind and although we take phenomena such as **reflection**, **refraction**, **diffraction** and **interference** for granted, it is not hard to see why they posed perplexing problems throughout most of history.

Why should light bend upon entering water?
Why does light spread out after passing through a narrow gap?
How does light travel to us from the sun, through the void of space?




- **Goal** Give necessary optics background to tackle a space mission, which includes an optical payload

OPTICS FOR REMOTE SENSING

I. General context

HISTORY

- These sorts of questions have ensured that optics has a long and engaging **history**.
- Mirrors were known to the ancients, eyeglasses were known by the thirteenth century, and, of course, the telescope was invented by **Galileo** around **1608**.
- The **law of refraction** was discovered by **Willebrord Snell** in **1621** and the **phenomenon of diffraction** was observed by both **Francesco Maria Grimaldi** and **Robert Hooke** by the **mid-1600s**.
- Sir **Isaac Newton** made great contributions to optics, proposing that '**white light**' was a combination of all colors, and formulating a particle, or corpuscular, theory of light.
- At roughly the same time (the latter half of the seventeenth century), the Dutch physicist **Christiaan Huygens** proposed a powerful **wave theory of light**.

- As we shall discover, most of the history of optics is dominated by the debate over the nature of light:

Is light a particle or a wave, or is it something in between (a wavicle?)?

In quantum mechanics, a **wavicle** is a wave-particle. An entity which simultaneously has the properties of a wave and a particle.

OPTICS FOR REMOTE SENSING

I. General context

HISTORY

- Another important figure in the history of optics is **Thomas Young**, an Englishman who revived the wave theory at the beginning of the nineteenth century by adding to it the principle of superposition.
- The French scientist **Augustin Jean Fresnel**, also an advocate of the wave theory, proposed a mechanistic description of light on the basis of it being a transverse oscillation through the ether, rather than a longitudinal one as had previously been assumed. The corpuscular theory seemed in very bad shape indeed.
- By **1845** **Michael Faraday** had performed several experiments showing that the plane of polarization could be altered by magnetic fields. This ultimately led to **James Clerk Maxwell**'s brilliant unification of optics and electromagnetism, when his wave equations predicted that the speed of light should be $1/\sqrt{\varepsilon_0 \cdot \mu_0}$, which was remarkably close to the experimental value. Light, then, was an electromagnetic disturbance propagating through the ether.

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OPTICS FOR REMOTE SENSING

I. General context

HISTORY

- As a wave, however, light must have a medium through which to propagate. Towards the end of the nineteenth century this medium, called the **ether**, became increasingly problematic; experiments by **Michelson** and **Morley** in particular could detect no motion of the ether relative to the earth.
- Such considerations led to **Einstein's theory** of special relativity and to the discarding of the idea of the ether altogether. Moreover, as the twentieth century progressed, quantum mechanics showed that all particles have a wavelike property; the distinction between waves and particles became less and less clear.
- In this lecture we will treat light usually as a **wave**, but sometimes as a **particle**, and as a general rule it is both or either.

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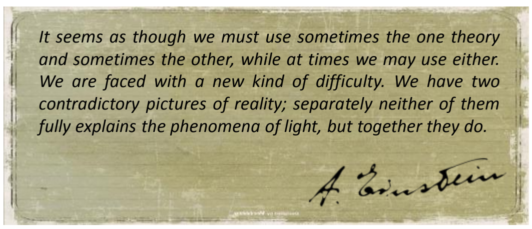
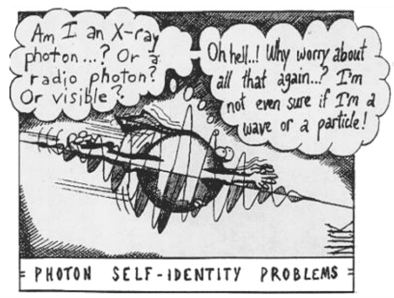
I. General context

- Light exhibits the quantum mechanical property of **wave - particle duality**.
- The **wave - particle duality** is the concept in quantum mechanics that every particle or quantum entity may be partly described in terms not only of particles, but also of waves. It expresses the inability of the classical concepts "particle" or "wave" to fully describe the behaviour of quantum-scale objects.
- We consider the **wave nature**, representing a probability distribution for photon location.
- We also consider the **particle interaction** of individual photons with matter.
- As Albert Einstein wrote:


It seems as though we must use sometimes the one theory and sometimes the other, while at times we may use either. We are faced with a new kind of difficulty. We have two contradictory pictures of reality; separately neither of them fully explains the phenomena of light, but together they do.

A. Einstein



I. General context

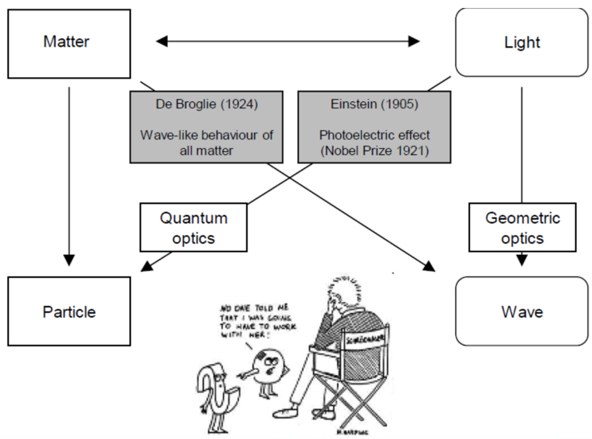
Photons
- Particles with energy *hv*
- Important for calculating detector efficiencies, considering scattering processes, beamsplitting, blackbody radiation

Waves
- With interference, vibrate charges in a substance have a polarization
- Important for diffraction limit, seeing, gratings, diffraction spikes, polarization effects in scattering

Rays
- Representing the path of photons (or equivalently wavefront normals)
- Important for thinking about optics, image quality, aberrations, etc. (not as fundamental as particle and waves, but useful analytically/geometrically)

I. General context

Geometrical *vs* **Physical Optics**



I. General context

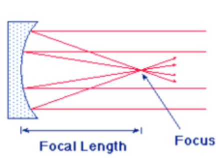
Geometrical *vs* **Physical Optics**

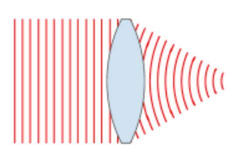
Geometrical Optics

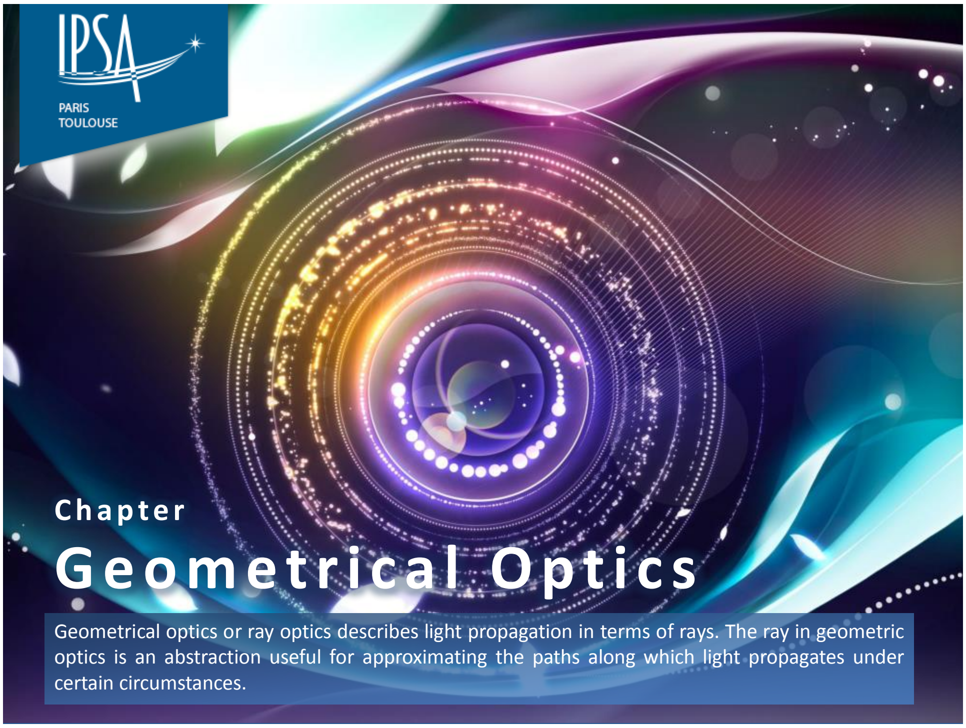
- Traces “rays” which do not interact with one another
- Purely Geometric (e.g. Snell’s law, law of reflection)
- Intersection of individual rays defines a focal plane and gives an image

Physical Optics

- Wave nature of light
- Wavefronts (rather than rays)
- Wave propagation and interference determine illumination
- Interference, diffraction, etc…







Chapter

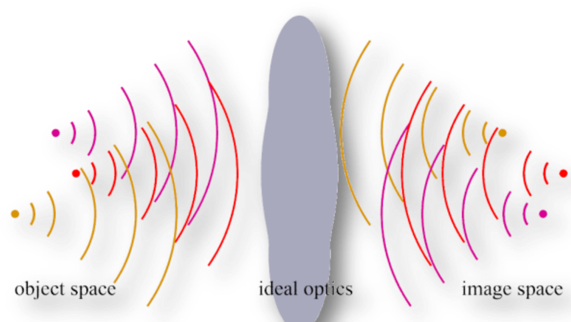
Geometrical Optics

Geometrical optics or ray optics describes light propagation in terms of rays. The ray in geometric optics is an abstraction useful for approximating the paths along which light propagates under certain circumstances.

II. Geometrical optics

GEOMETRICAL APPROXIMATION

- **Ideal optics** = spherical waves from any point in object space are imaged into points in image space
- Corresponding points are called **conjugate points**
- **Focal point** = center of converging or diverging spherical wavefront
- Object space and image space are reversible

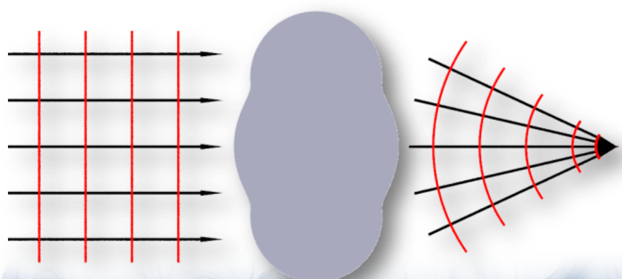


OPTICS FOR REMOTE SENSING

II. Geometrical optics

GEOMETRICAL APPROXIMATION

- Rays are normal to locally flat wave (locations of constant phase)
- Rays are reflected and refracted according to Fresnel equations
- Phase is neglected ⇨ Incoherent sum
- Rays can carry polarization information
- Optical system is finite ⇨ Diffraction effects
- Geometrical optics neglects diffraction effects: $\lambda \rightarrow 0$
- Physical optics $\lambda > 0$
- Simplicity of geometrical optics mostly outweighs limitations



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II. Geometrical optics

FUNDAMENTALS OF LIGHT

Fundamentals of light

- Light travels in a **straight line** in constant - refractive - index medium at speed c/n
- Refractive index n is 1.0 in vacuum, and is related to the permittivity (ε) and permeability (μ) of material

$$\frac{1}{c} = \sqrt{\varepsilon_0 \cdot \mu_0} \qquad \frac{1}{v} = \sqrt{\varepsilon \cdot \mu} \qquad n = \sqrt{\frac{\varepsilon \cdot \mu}{\varepsilon_0 \cdot \mu_0}}$$

Material	Index (n)	notes
air	1.000274	274ppm different from vacuum
water	1.333	Similar for eyeball
quartz	1.458	Aka, fused silica
BK7 glass	1.52	Common optics for lenses
Diamond	2.419	
Silicon	~3.5	CCD material, for instance

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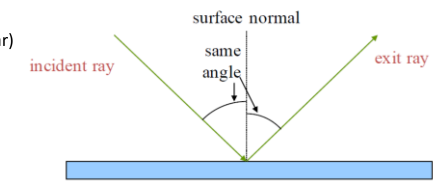
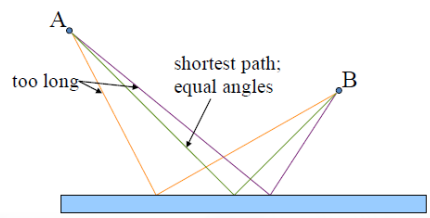
II. Geometrical optics

Reflection

- Reflection off a flat surface follows a simple rule:
  - Angle of incidence equals angle out
  - Angles measured from surface "normal" (perpendicular)



- Also consistent with the **principle of least time**:
  - Fermat's principle
  - If going from point A to point B, reflecting off a mirror, the path traveled is also the **most expedient** (shortest) route



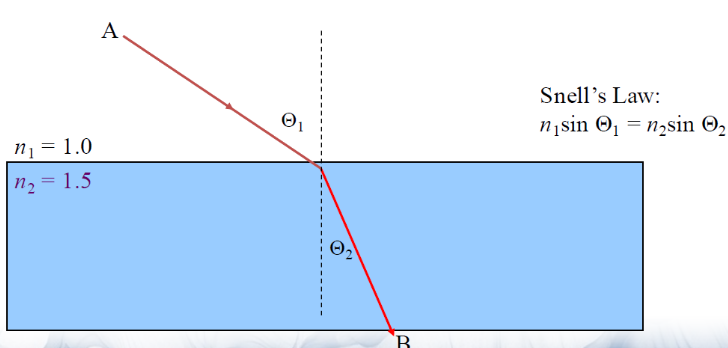
OPTICS FOR REMOTE SENSING

II. Geometrical optics

FUNDAMENTALS OF LIGHT

Refraction

- Light bends at interface between refractive indices (n)
  - Bends more the larger the difference in refractive index
  - Can be effectively viewed as a **least time** behavior – get from A to B faster if you spend less time in the slow medium



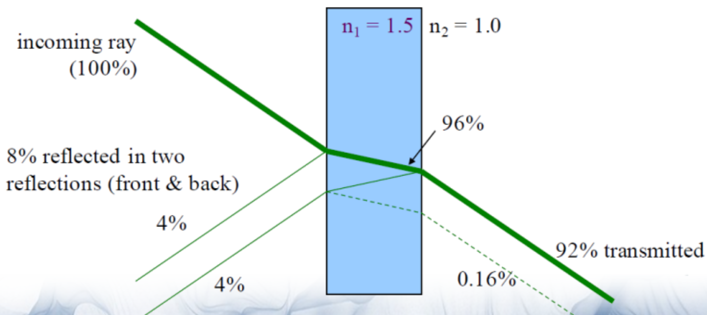
II. Geometrical optics

Optical design considering every surface

- Let's consider a thick piece of glass ($n = 1.5$) and the light paths associated with it. We define the reflection fraction

$$R = \left[\frac{n_1 - n_2}{n_1 + n_2}\right]^2$$

Using $n_1 = 1.5$ and $n_2 = 1.0$ (air) ⇨ $R = \left(\frac{0.5}{2.5}\right)^2 = 0.04 = 4\%$



II. Geometrical optics

II. Geometrical optics

Lens definition

- A **lens** is a transmissive optical device that focuses or disperses a light beam by means of refraction.

- A **simple lens** consists of a single piece of transparent material, while a **compound lens** consists of several simple lenses (elements), usually arranged along a common axis.
- Lenses are made from materials such as **glass** or **plastic**, and are ground and polished or molded to a desired shape.
- A lens can focus light **to form an image**, unlike a prism, which refracts light without focusing.
- Devices that similarly **focus** or disperse waves and radiation other than visible light are also called **lenses**, such as microwave lenses, electron lenses, acoustic lenses, or explosive lenses.

Compound lens including several simple lenses or elements

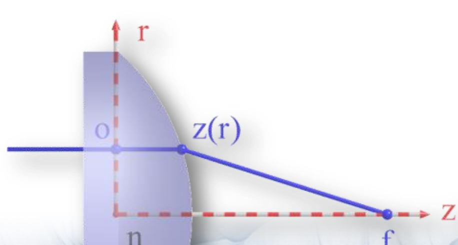
OPTICS FOR REMOTE SENSING

II. Geometrical optics

LENSES

Surface shape of perfect lens

- Lens material has **index of refraction *n***
- $\overline{oz(r)} \cdot n + \overline{z(r)f} = constant$
- $n \cdot z(r) + \sqrt{r^2 + \left(f - z(r)\right)^2} = constant$
- Solution $z(r)$ is an hyperbola with an eccentricity $e = n > 1$

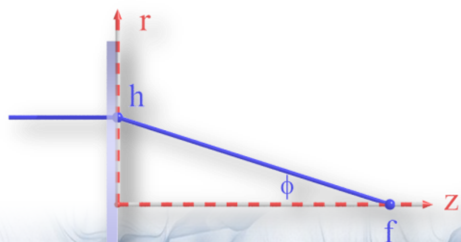


II. Geometrical optics

LENSES

Paraxial optics and Assumptions

- Snell's law for small angles of incidence $\sin(\phi) \sim \phi$
- Ray height h is small so that optics curvature can be neglected (plane optics) $\cos(\phi) \sim 1$
- $\tan(\phi) \sim \phi = h/f$
- These assumptions are decent until about 10 degrees



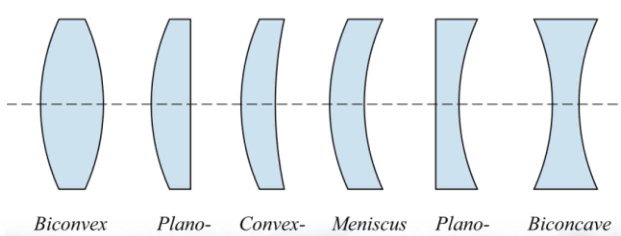
OPTICS FOR REMOTE SENSING

II. Geometrical optics

LENSES

Spherical lenses

- If two spherical surfaces have same the radius, one can fit them together
- Surface error requirement less than $\lambda/10$
- Grinding spherical surfaces is easy ⇨ most optical surfaces are spherical
- System miniaturization: **Freeform optics** involve optical designs with at least one freeform surface which, according to the ISO standard 17450-1:2011, has no translational or rotational symmetry about axes normal to the mean plane.



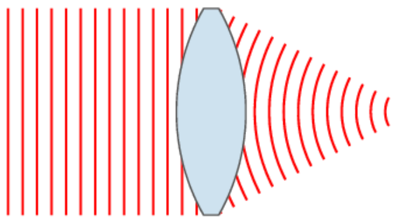
II. Geometrical optics

LENSES

Positive / converging spherical lens parameters

- Center of curvature and radii with signs — $R_1 > 0$ and $R_2 < 0$
- Center thickness — d
- Positive focal length — $f > 0$

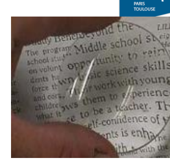
Incident rays parallel to the optical axis **converge** toward the far focal point



OPTICS FOR REMOTE SENSING

II. Geometrical optics

LENSES



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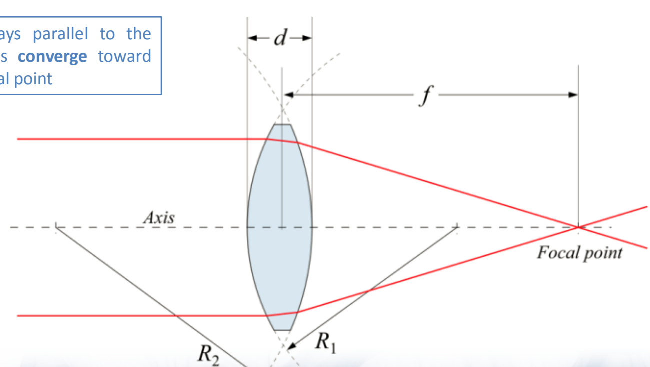
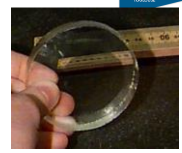
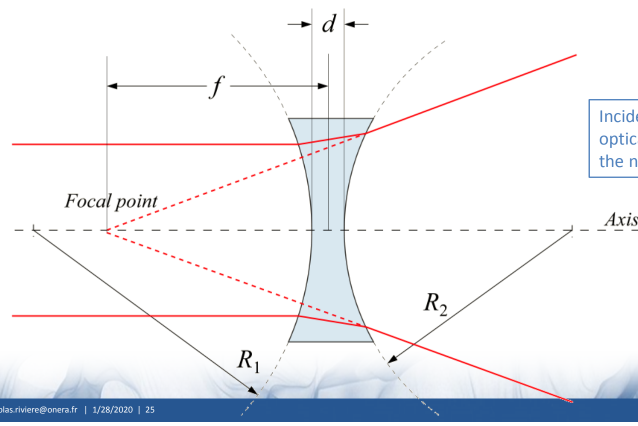


II. Geometrical optics

Negative / diverging spherical lens parameters



- Note different signs of radii $R_1 < 0$ and $R_2 > 0$
- Virtual focal point
- Negative focal length $f < 0$

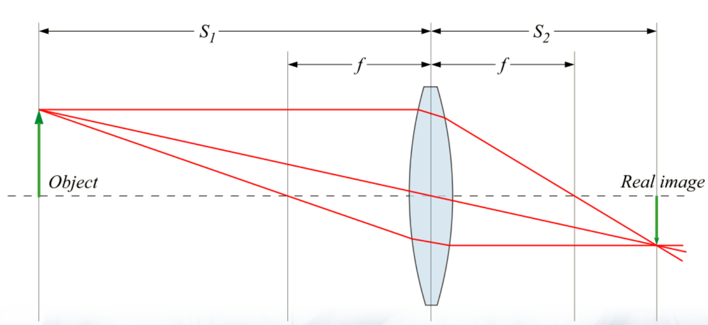


Incident rays parallel to the optical axis **diverge** away from the near focal point

II. Geometrical optics

General lens setup and Real image

- Object distance S_1 Object height h_1
- Image distance S_2 Image height h_2
- Axis through two centers of curvature is called **optical axis**
- Surface point on optical axis is called the **vertex**
- **Chief ray** through center maintains direction



OPTICS FOR REMOTE SENSING

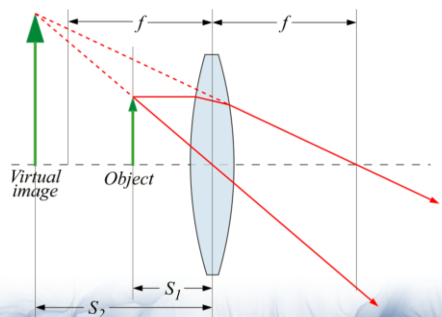
II. Geometrical optics

LENSES

General lens setup and Virtual image

- Object distance S_1 Object height h_1
- Image distance S_2 Image height h_2

- Note that the object is closer than the focal length of lens
- Virtual image



OPTICS FOR REMOTE SENSING

II. Geometrical optics

LENSES

Thin lens approximation

- Thin-lens equation $$\boxed{\frac{1}{S_1}+\frac{1}{S_2}=(n-1)\cdot\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

- Gaussian lens formula $$\boxed{\frac{1}{S_1}+\frac{1}{S_2}=\frac{1}{f}}$$

Finite imaging

- Rarely image point sources, but extended object
- Object and image size are proportional
- Orientation of object and image are inverted
- (transverse) magnification perpendicular to optical axis $$\boxed{M=\frac{h_2}{h_1}=-\frac{S_2}{S_1}}$$

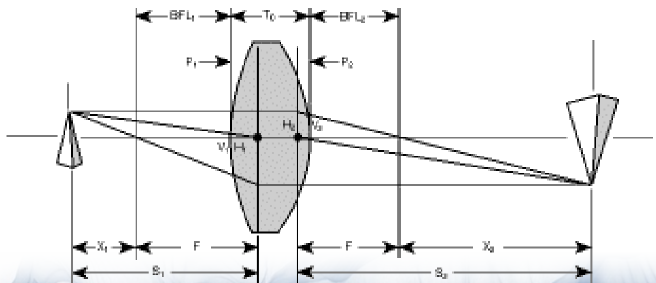
II. Geometrical optics

Thick lenses

- Basic thick lens equation

$$\frac{1}{f} = (n-1)\cdot\left(\frac{1}{R_1} - \frac{1}{R_2} + \boxed{\frac{(n-1)\cdot d}{n\cdot R_1\cdot R_2}}\right)$$

- Reduces to thin lens for $d \ll R_1 \cdot R_2$

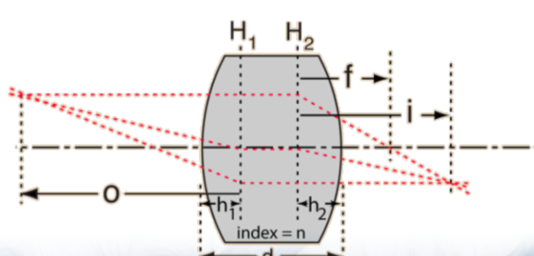


II. Geometrical optics

Principal planes

- Focal lengths and distances are measured from the **principal planes**
- Refraction can be considered to only happen at principle planes
- Principle planes only depend on lens properties itself
- Thin lens equation neglects finite distance between principle planes
- Distance between vertices and principal planes is given by

$$H_{1,2} = -\frac{f\cdot(n-1)\cdot d}{R_{2,1}\cdot n}$$



OPTICS FOR REMOTE SENSING

II. Geometrical optics

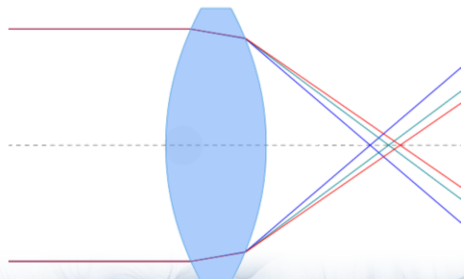
LENSES

Chromatic aberration

- Chromatic aberration is caused by the dispersion of the lens material *ie* the variation of its refractive index *n*, with the wavelength of light.

- Since, from the formulae above, *f* is dependent upon *n*, it follows that light of different wavelengths is focused to different positions.

- Chromatic aberration is due to **wavelength dependence of index of refraction**
- Higher index in the blue ⇨ shorter focal length in blue



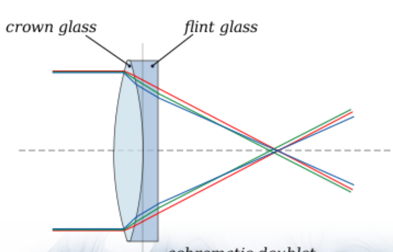
OPTICS FOR REMOTE SENSING

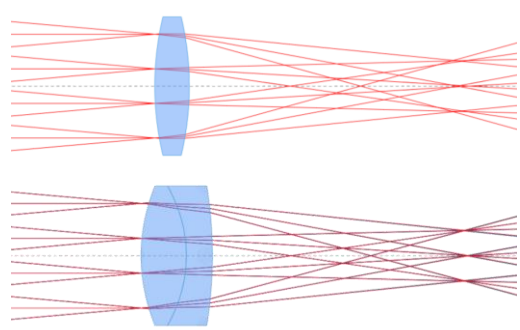
II. Geometrical optics

LENSES

Achromatic doublet

- Combination of 2 lenses with different glass dispersion
- Also less spherical aberration





II. Geometrical optics

MIRRORS

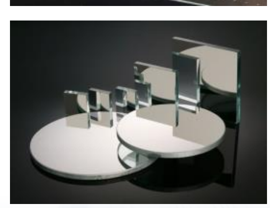
MIRRORS

II. Geometrical optics

MIRRORS

Mirrors *vs* Lenses

- Mirrors are completely achromatic
- Reflective over very large wavelength range (UV to radio)
- Can be supported from the back
- Can be segmented
- Wavefront error is twice that of surface, lens is (n-1) times surface
- Only one surface to 'play' with



II. Geometrical optics

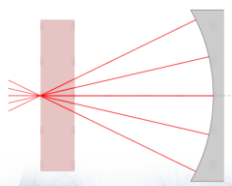
Spherical mirrors

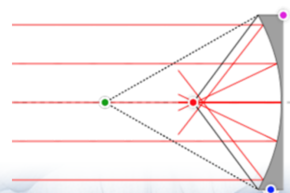
- Curved mirrors come in two basic types: those that converge parallel incident rays of light and those that diverge parallel incident rays of light.

- One of the easiest shapes to analyze is the **spherical mirror**. Such a mirror is not a complete sphere, but a spherical cap — a piece sliced from a larger imaginary sphere with a single cut.

- Easy to manufacture
- Focuses light from center of curvature onto itself
- Focal length is half of curvature
- Tip-tilt misalignment does not matter
- Has no optical axis
- Does not image light from infinity correctly: **spherical aberration**

$$f = R/2$$





II. Geometrical optics

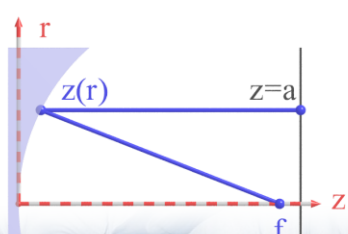
MIRRORS

Parabolic mirrors

- A **parabolic** (or paraboloid or paraboloidal) **reflector** (or dish or mirror) is a reflective surface used to collect or project energy such as light, sound, or radio waves.

- Its shape is part of a **circular paraboloid** (surface generated by a parabola revolving around its axis). The parabolic reflector transforms an incoming plane wave traveling along the axis into a spherical wave converging toward the focus.

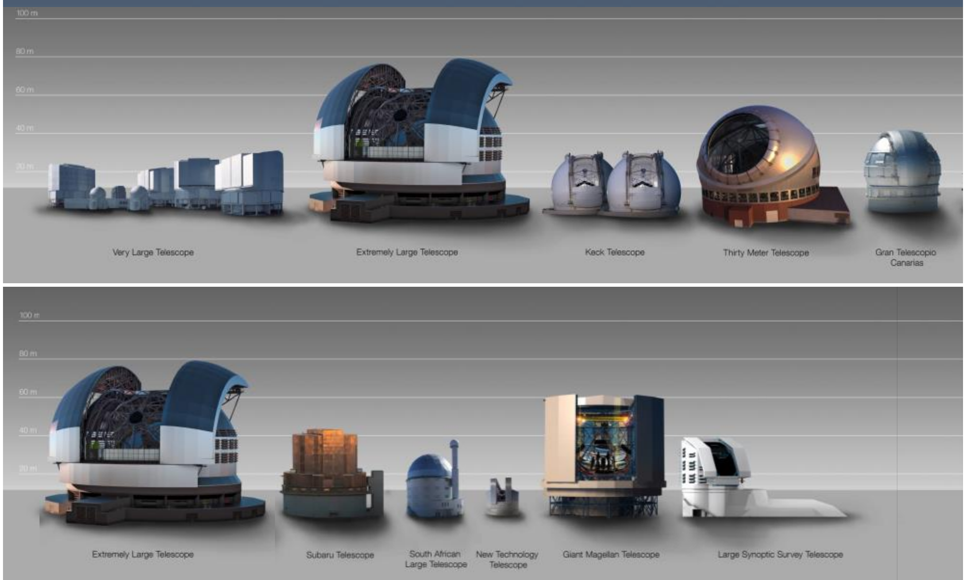
- Want to make flat wavefront into spherical wavefront
- Distance $\overline{az(r)} + \overline{z(r) \cdot f} = constant$ and $z(r) = r^2/2 \cdot R$
- Perfect image of objects at infinity
- Has clear optical axis



OPTICS FOR REMOTE SENSING

II. Geometrical optics

MIRRORS

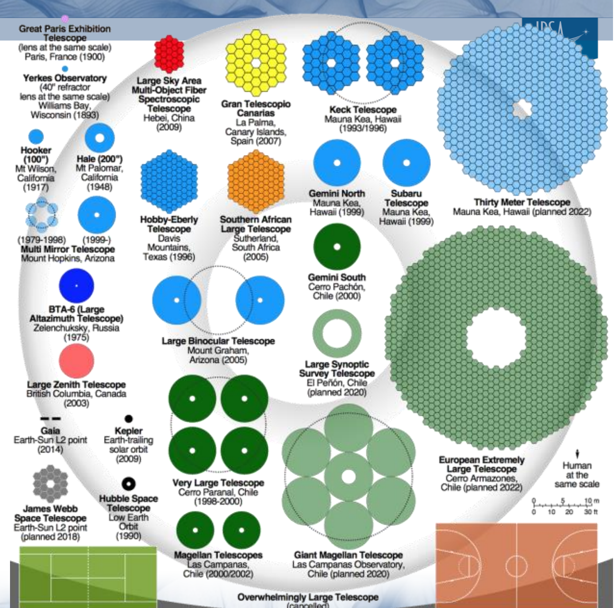


OPTICS FOR REMOTE SENSING

II. Geometrical optics

MIRRORS

Telescope mirror comparison

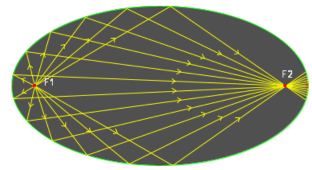


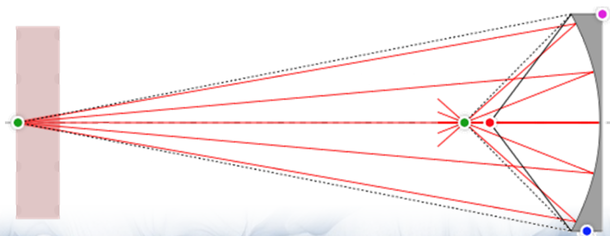
II. Geometrical optics

Elliptical mirrors

- The **elliptical mirrors** have two focal points: light that passes through one of these before striking the mirror is reflected such that it passes through the other.

- Have two foci at finite distances
- Perfectly reimage one focal point into another





OPTICS FOR REMOTE SENSING

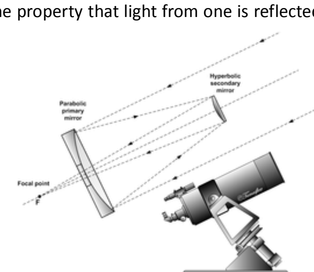
II. Geometrical optics

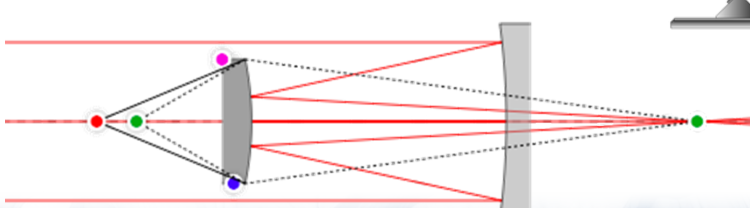
MIRRORS

Hyperbolic mirrors

- The focus of a **hyperbolic mirror** is either of two points which have the property that light from one is reflected as if it came from the other.

- Have a real focus and a virtual focus (behind mirror)
- Perfectly reimage one focal point into another





II. Geometrical optics

OPTICAL SYSTEMS

OPTICAL SYSTEMS

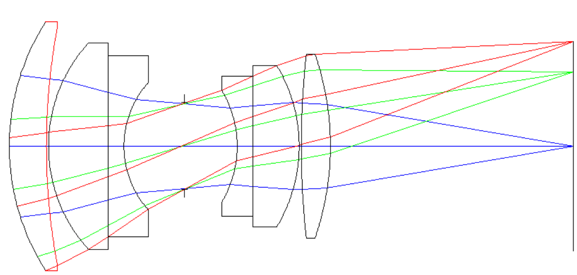
II. Geometrical optics

OPTICAL SYSTEMS

Overview

- Combinations of several optical elements (lenses, mirrors, stops)
- Examples: camera “lens”, microscope, telescopes, instruments
- Thin-lens combinations can be treated analytically
- Effective focal length:

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$$



II. Geometrical optics

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Overview

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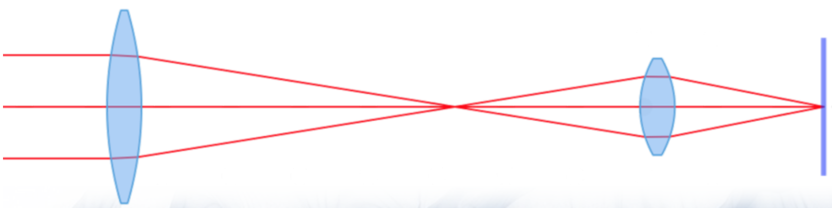
Simple thin-lens combinations

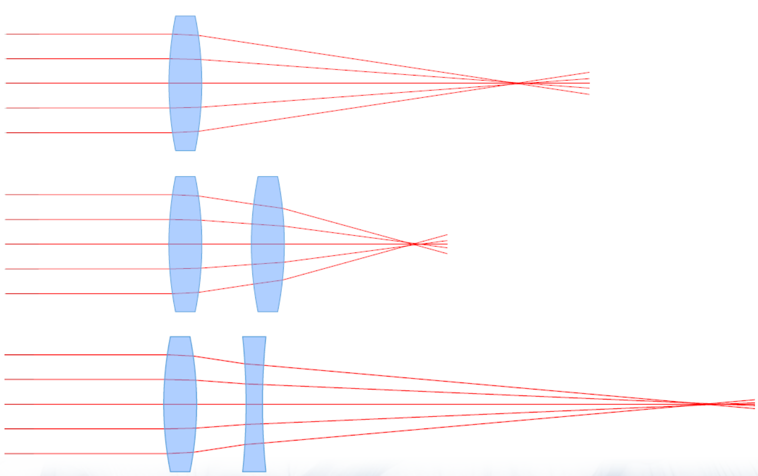
- Distance > sum of focal lengths ⇨ real image between lenses
- Apply single-lens equation successively



II. Geometrical optics

OPTICAL SYSTEMS

Second lens adds convergence or divergence



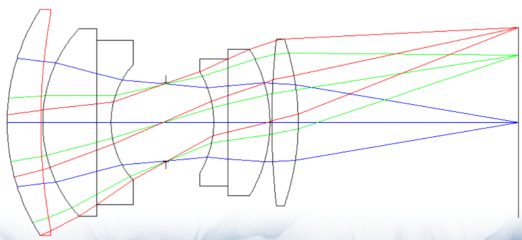
II. Geometrical optics

F-number and numerical aperture

- In optics, an **aperture** is a hole or an opening through which light travels.
- More specifically, the aperture and focal length of an optical system determine the **cone angle of a bundle of rays** that come to a focus in the image plane.

- All optical systems have a place where 'aperture' is limited
- Main mirror of telescopes
- Aperture stop in photographic lenses
- Aperture typically has a maximum diameter
- Aperture size is important for diffraction effects

Aperture mechanism of Canon 50mm f/1.8 II lens, with five blades



OPTICS FOR REMOTE SENSING

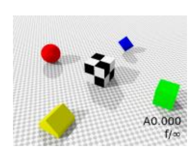
II. Geometrical optics

OPTICAL SYSTEMS

F-number and numerical aperture

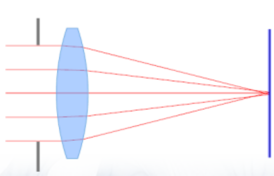
- The **f-number** of an optical system (such as a camera lens) is the ratio of the system's focal length to the diameter of the entrance pupil.
- It is a **dimensionless number** that is a quantitative measure of lens speed, and an important concept in photography. It is also known as the **focal ratio**, **f-ratio**, or **f-stop**.
- It is the reciprocal of the relative aperture. The f-number is commonly indicated using a hooked *f* with the format *f/D*, where *D* is the f-number.

- Describes the light-gathering ability of the lens
- The **f-number** given by $F = f/D$
- Also called focal ratio or f-ratio, written as: f/F
- The bigger *F*, the better the paraxial approximation works
- Fast system for *F* < 2, slow system for *F* > 2

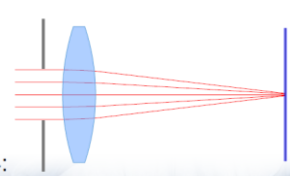


Computer simulation showing the effects of changing a camera's aperture from zero to infinity

f/2:



f/4:



II. Geometrical optics

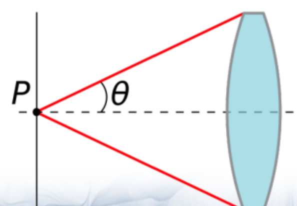
F-number and numerical aperture

- In optics, the **numerical aperture** (NA) of an optical system is a dimensionless number that characterizes the range of angles over which the system can accept or emit light.
- By incorporating index of refraction in its definition, NA has the property that it is **constant for a beam** as it goes from one material to another, provided there is **no refractive power** at the interface.

- Numerical aperture definition

$$NA = n \cdot \sin(\theta)$$

- *n* index of refraction of working medium
- θ is the half-angle of maximum cone of light that can enter or exit the lens
- This notion is important for microscope objectives (*n* often not 1)



OPTICS FOR REMOTE SENSING

II. Geometrical optics

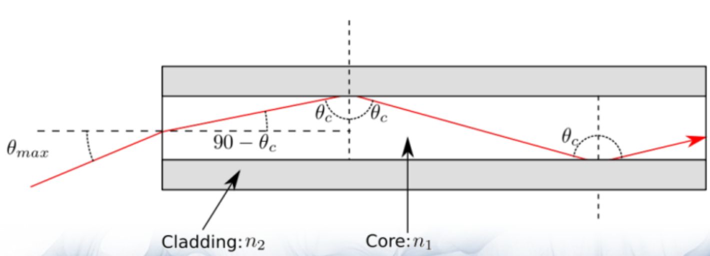
OPTICAL SYSTEMS

F-number and numerical aperture

- The **numerical aperture in Fibers** is the acceptance cone of the fiber determined by the materials

- In that case, numerical aperture definition is given by
- *n* index of refraction of working medium

$$NA = n \cdot \sin(\theta) = \sqrt{n_1^2 - n_2^2}$$



II. Geometrical optics

OPTICS FOR REMOTE SENSING

II. Geometrical optics

IMAGES & PUPILS

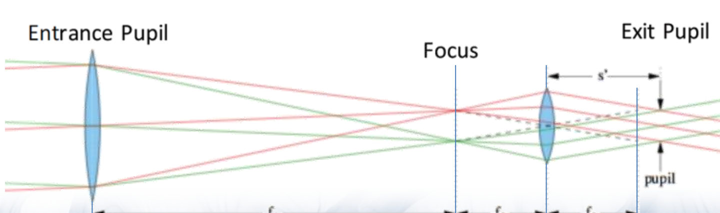
Image definition

- Every object point comes to a focus in an image plane
- Light in one image point comes from pupil positions
- Object information is encoded in position, not in angle

Pupil definition

- All object rays are smeared out over complete aperture
- Light in one pupil point comes from different object positions
- Object information is encoded in angle, not in position

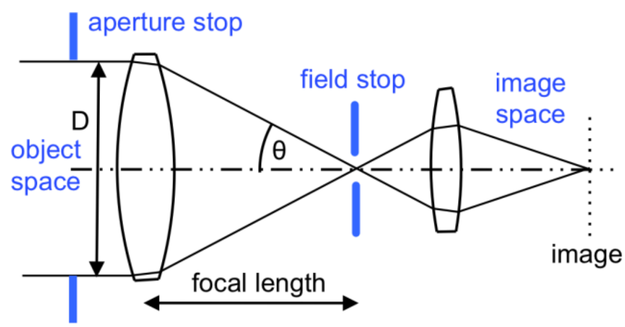
- **Pupils** are images of physical stops
- **Entrance pupil**: the image of the aperture stop looking forward from object space formed by the intervening lenses
- **Exit pupil**: the image of the aperture stop looking back from image space formed by the intervening lenses.



II. Geometrical optics

Aperture and field stops

- Aperture stop limits the amount of light reaching the image
- Aperture stop determines light-gathering ability of optical system
- Field stop limits the image size or angle



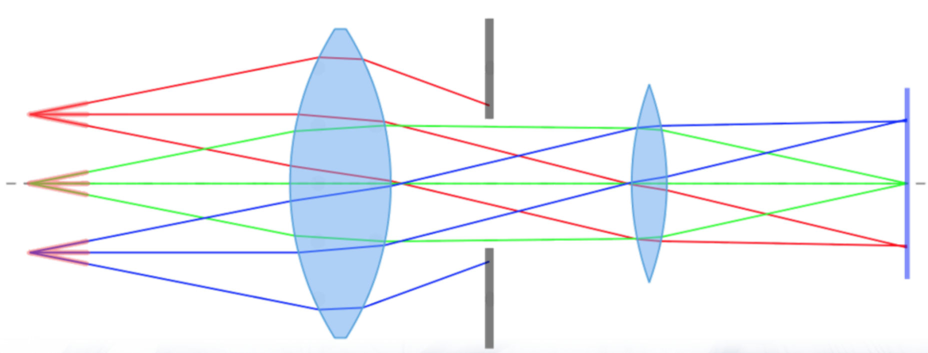
OPTICS FOR REMOTE SENSING

II. Geometrical optics

IMAGES & PUPILS

Vignetting

- Effective aperture stop depends on position in object
- Image fades toward its edges



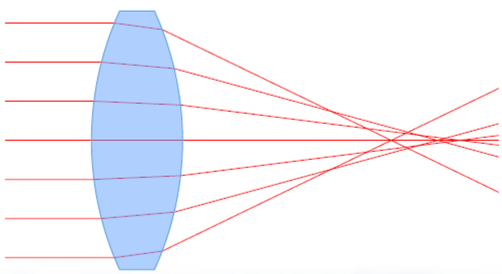
OPTICS FOR REMOTE SENSING

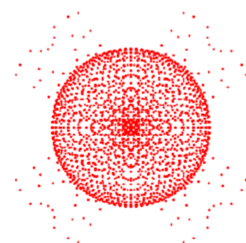
II. Geometrical optics

IMAGES & PUPILS

Spherical aberration of spherical lens

- Different focal lengths of paraxial and marginal rays
- Longitudinal spherical aberration along optical axis
- Transverse (or lateral) spherical aberration in image plane
- Much more pronounced for short focal ratios
- Foci from paraxial beams are further away than marginal rays
- Spot diagram shows central area with fainter disk around it





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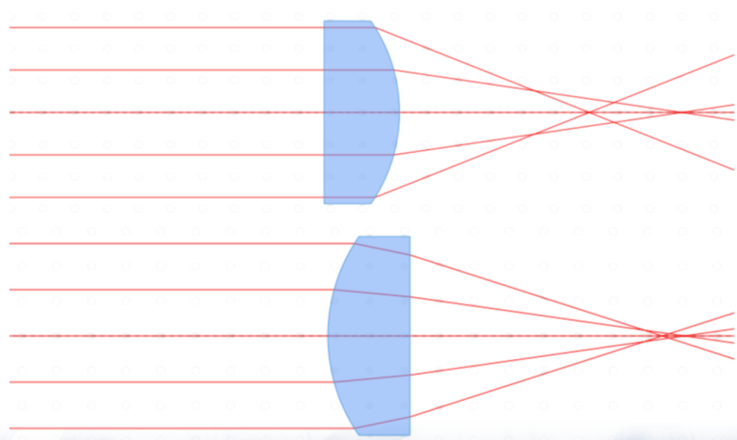
OPTICS FOR REMOTE SENSING

II. Geometrical optics

IMAGES & PUPILS

Spherical aberration of spherical lens

- Minimizing spherical aberrations?



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OPTICS FOR REMOTE SENSING

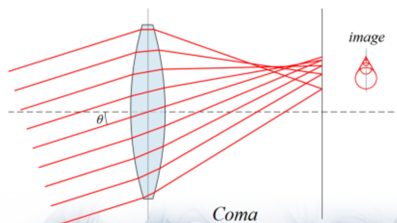
II. Geometrical optics

IMAGES & PUPILS

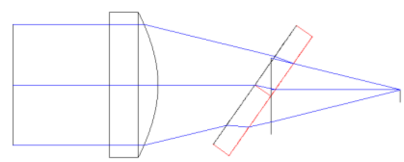
Other possible aberrations

- In optics (especially telescopes), the **coma** or **chromatic aberration** in an optical system refers to aberration inherent to certain **optical designs** or due to **imperfection** in the lens or other components that results in **off-axis point sources** such as stars appearing distorted, appearing to have a tail (coma) like a comet.

- The **astigmatism** is due to **Tilted Glass Plate** in converging beam. It focuses in two orthogonal directions, but not in both at the same time.



Coma


Astigmatism

OPTICS FOR REMOTE SENSING

II. Geometrical optics

IMAGES & PUPILS

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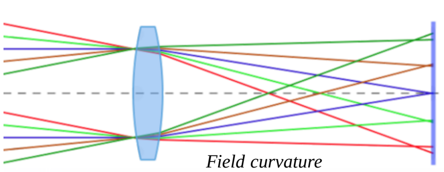
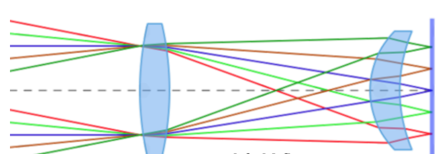
- Petzval or **field curvature**: image lies on curved surface
- Petzval field flattening: Petzval curvature only depends on index of refraction and focal length of lenses. It is independent of lens position!


Field curvature


Petzval field flattering

II. Geometrical optics

Distorsion

- In geometric optics, **distortion** is a deviation from rectilinear projection: a projection in which straight lines in a scene remain straight in an image. It is a form of optical aberration.

- Image is sharp but geometrically distorted

(a) object
(b) positive (or pincushion) distortion
(c) negative (or barrel) distortion